COST THEORY

Cost theory is related to production theory, they are often used together. However, the question is usually how much to produce, as opposed to which inputs to use. That is, assume that we use production theory to choose the optimal ratio of inputs (eg. 2 fewer engineers than technicians), how much should we produce in order to minimize costs and/or maximize profits? We can also learn a lot about what kinds of costs matter for decisions made by managers, and what kinds of costs do not.

I What costs matter?

A Opportunity Costs

Remember from Section (IV) of the Introduction, that in addition to accounting profit, managers must consider the cost of inputs supplied by the owners (owners capital and labor).

Definition 17 Explicit Costs: *Accounting Costs, or costs that would appear as costs in an accounting statement.*

Definition 18 Opportunity Costs: *The value of all inputs to a firm's production in their most valuable alternative use.*

Recall the example from Section (IV), where the decision was whether or not to buy the kitchen. the opportunity costs were the $24,000 per year the money could be earning elsewhere, and the owners time cost of $40,000, which exceeded the accounting profit of $60,000.

B Fixed costs, variable costs, and sunk costs

Some inputs vary with the amount produced and others do not. The firm's computer system and accountants may be able to handle a large volume of sales without increasing the number of computers or accountants, for example. Inputs that do not vary with the amount produced, like accountants and computers, are called fixed inputs.

Most inputs are fixed only for a certain range of production. A medical office may be able to handle many additional patients without adding and office assistant or extra phones. The phones and office assistants are fixed inputs. But, if the number of extra patients is large enough, the firm needs extra office staff.

Reasons for fixed costs:

1. **Salaried workers**. Salaried workers are a fixed input if the worker can work overtime without additional compensation (doctors paid via fee for service are variable inputs, salaried medical staff are fixed inputs).

2. **Fixed hours at work**. An hourly worker sometimes cannot be sent home early if not enough work is available. Therefore, workers may not be busy and be able to handle extra work without additional hours.

3. **Time to adjust.** Some inputs, like machines, take time to purchase and install. Conversely, unskilled labor may be adjusted more quickly through overtime, temps, etc. Therefore, by necessity the firm may only be able to vary production by increasing labor in the short run.

Definition 19 Total Variable Cost *The total cost of all inputs that change with the amount produced (all variable inputs).*

Definition 20 Total fixed costs *The total cost of all inputs that do not vary with the amount produced (all fixed inputs).*

Consider the Thompson machine company. The firm uses 5 machines to make machine parts. Because of the time to adjust, machines are a fixed cost, while the number of workers varies with the amount produced. Labor is a variable cost.

Definition 21 Sunk costs *Are costs that have been incurred and cannot be reversed.*

Any costs incurred in the past, or indeed any fixed cost for which payment must be made regardless of the decision is irrelevant for any managerial decision. Suppose you hire an executive with a $100,000 signing bonus, plus $200,000 salary. After hiring, you may find the executive does not live up to expectations. However, if the executive's marginal revenue product is $200,001, the executive still generates $1,000 in profits relative to his salary and therefore should be retained. But if his MRP is $199,999, the firm loses an extra $1,000 each year they keep him, so he should be let go. The bonus is a sunk cost and does not affect the retention decision.

The principle of sunk costs is equivalent to the saying "don't throw good money after bad."

Sometimes a decision can be made to recover part of a fixed cost. Perhaps one could sell a factory and recover part of the fixed costs. Then only the difference is sunk. For example, if we can sell a building for which we paid $500,000 for $300,000, then only $200,000 is sunk.

Sunk costs are perhaps one of the most psychologically difficult things to ignore. Examples:

1. **Finance**. Studies show investors let sunk costs enter their decision making. What price the stock was purchased at is sunk and therefore irrelevant. What matters is only whether or not this stock offers the best return for the risk. Yet, investors are reluctant to sell stocks whose price has gone down.

2. **Capital investment**. I watched the world series of poker. In one instance, the odds of drawing a flush (and almost for sure winning the hand) was 1 in 5. The pot was about $200,000. So the player should call any bet less than or equal to $40,000. Yet the commentator advised that the player should call regardless of the bet, because he already had so much money in the pot (sunk costs).

3. **Cut your losses**? Consider the war in Afghanistan. We have sunk billions, but that should not enter our decision about whether or not to stay.

4. **Pricing in high rent districts**. Consider restaurants in a high rent district (say an airport). Should they take the rent into account when setting prices? No. In fact, prices are high not because of the rent, but typically because of the lack of competitors.

II Short run costs

We use short run costs primarily to compute how much to produce while maximizing profits. We use long run costs to answer questions like should the firm expand, contract, merge, etc.

Definition 22 Average Costs: *Costs divided by output.*

Definition 23 Marginal Costs: *The cost of one additional unit of an input.*

Here is the notation:

Type of Cost	Total Cost equals	Variable Costs	Plus Fixed Costs
Total	$TC =$	TVC	$+TFC$
Average	$ATC = \frac{TC}{Q} =$	$AVC = \frac{TVC}{Q}$	$+AFC = \frac{TFC}{Q}$
Marginal	$MC = \frac{dTC}{dQ} = \frac{\Delta TC}{\Delta Q}$		

Properties of cost functions in the short run:

1. Total costs of course increase with Q, the quantity produced.

2. Average Costs decline with Q, but eventually rise. The fixed costs are spread over many more units of production at high Q, reducing average costs. All of the extra workers required for producing additional units when the factory is near capacity starts to increase average costs eventually.

3. Marginal costs usually decline then increase, but must eventually increase. At first, producing one additional unit is may cheaper than the last unit, due to specialization. However, eventually diminishing returns sets in and the workers just get in each other's way. Then a very large number of additional workers might be needed to produce an additional unit.

Here is a graph of the cost curves.

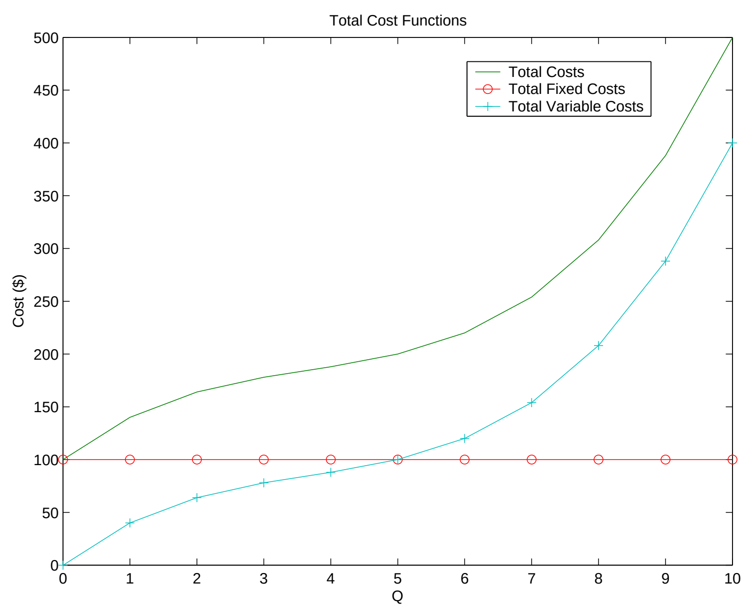

Figure 5: Typical short total cost curves.

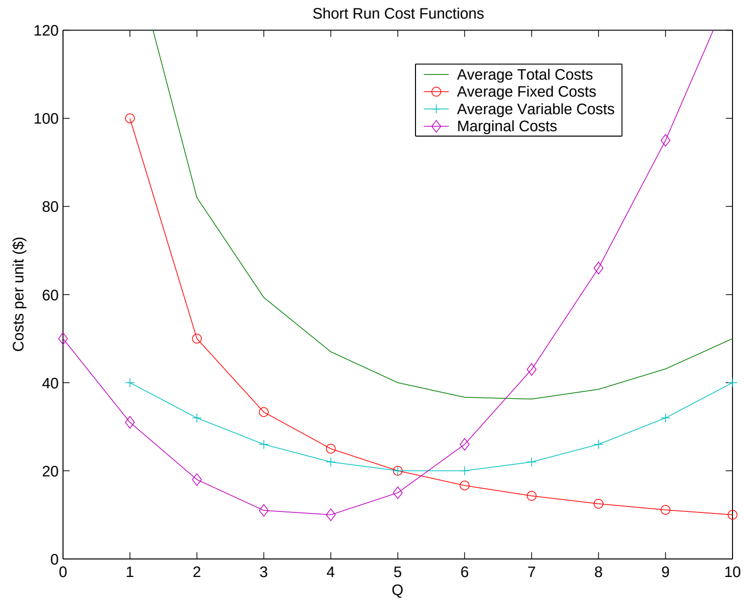

Figure 6: Typical short run average and marginal cost curves.

III Examples using short run cost curves.

A Profit maximization with perfect competition

Let us suppose that you are a hypothetical manager of a group of cruise ships. Using data from previous years, you estimate the short run cost function is (we will see how to do this estimation below):

$$TC = 60 + \frac{Q^2}{20} \tag{71}$$

Here Q is the number of cruises the ship takes (not the number of passengers). The cost of the ship is \$60 million, which is sunk. Notice in equation (71) that the cost of the ship does not depend on the number of cruises the ship takes. Suppose further that each cruise generates revenue of \$3 million.

Maximize profits:

$$\max \pi = TR - TC = 3Q - 60 - \frac{Q^2}{20} \tag{72}$$

Take the derivative to get the slope and set the slope equal to zero:

$$3 - \frac{Q}{10} = 0 \quad \rightarrow \quad Q = 30. \tag{73}$$

Notice that the fixed costs have dropped out. The math agrees: fixed costs do not matter for our decision. In general, to maximize profits we set marginal revenue (here \$3) equal to marginal costs (here $Q/10$).

$$MR = MC. \tag{74}$$

Using a table:

Cruises (Q)	Total variable costs (TVC)	Marginal costs (MC)
0	0	–
20	$= (20^2)/20 = 20$	$= (20-0)/(20-0) = 1$
25	$= (25^2)/20 = 31.25$	$= (31.25-20)/(25-20) = 2.25$
30	45	2.75
35	61.25	3.25
40	80	3.75

Table 5: Variable costs on a cruise ship.

The marginal revenue is the price of the cruise, equal to \$3 million. We can see that marginal revenue equals marginal costs somewhere between 30 and 35 cruises.[2]

Producing the 30th cruise gives us \$3 of revenue, enough to cover the costs of producing the 30th cruise, which (using table 5) is approximately \$2.75. However, the 35th cruise loses money. The table estimates that cruise would cost \$3.25, and since revenues are \$3, the cruise would lose an estimated \$0.25 million.

The fixed costs are irrelevant here. When considering the fixed costs, the firm has negative profits regardless of how many cruises the firm takes. The maximum profits occurs at 30 cruises:

$$\pi = TR - TC = 3 \cdot 30 - \left(60 + \frac{30^2}{20}\right) = -\$15. \tag{75}$$

We have already paid the fixed costs, so we might as well lose as little as possible.[3]

[2]The table is an approximation, however. In fact, using the true marginal cost of the 30th cruise is $MC = dTC/dQ = Q/10 = 3$.

[3]Note that it is irrelevant how the ship is financed. Interest and principle payments are also sunk costs.

B Break Even Analysis

An important consideration when deciding whether to continue operations in a particular market, expand into a market, or start a new business is a break even analysis. We can do a break even analysis with a cost function.

In a break even analysis, the question is how much profit is required to exactly pay off all fixed costs. Alternatively, how much revenue is required to pay off the average variable costs and the fixed costs:

$$\pi = 0 = TR - TC \tag{76}$$

$$0 = P \cdot Q - TFC - TVC \tag{77}$$

$$0 = P \cdot Q - TFC - AVC \cdot Q \tag{78}$$

$$Q = \frac{TFC}{P - AVC} \tag{79}$$

Here I have assumed linear total costs, so that average variable cost is constant. One could assume (more realistically) that average variable costs depend on Q, and use a table to get the break even point.

C Average Costs

Often average cost data is easy to get. It is relatively easy to measure total costs and the quantity sold to get average costs. However, many managers then incorrectly base decisions on average costs.

Consider the following data:

Typical Data			Calculate this!	
Gas Production (Q)	Total costs (TC)	Average costs (AC)	Marginal Costs (MC)	Profits (π)
20	1271	$= 1271/20 = 63.6$	–	89
22	1359	61.8	$= (1359 - 1271)/(22 - 20) = 44.0$	137
24	1456	60.7	48.5	176
26	1562	60.1	53.0	206
28	1675	59.8	56.5	229
30	1797	59.9	61.0	243
32	1928	60.3	65.5	248
34	2067	60.8	69.5	245
36	2214	61.5	73.5	234
38	2370	62.4	78.0	214
40	2534	63.4	82.0	186
42	2707	64.5	86.5	149
44	2888	65.6	90.5	104
46	3077	66.9	94.5	51
48	3275	68.2	99.0	-11

Table 6: Average and marginal costs in the gas industry.

The first 3 columns of table 6 give a typical data set one might have in a real business situation. For example, a gas refinery might produce different quantities monthly depending on variations in the price of gas and demand. Suppose the price of gas is currently \$68. How much gas should be produced? Setting price equal to average cost (48 units) actually produces a loss. We could try to minimize average costs, which occurs at 28 units. Still, this is not the maximum profits. The maximum profits occurs when price equals marginal costs, about 32-34 units. The key is that given the first 3 columns, one can easily calculate the marginal costs and calculate the profit maximizing quantity.

IV Long Run Costs

We use long run costs to decide scale issues, for example mergers. We assume the long run is long enough for all costs to be variable.

In the long run, we can build any size factory we wish, based on anticipated demand, profits, and other considerations. Once the plant is built, we move to the short run as described above. Therefore, it is important to forecast the anticipated demand. Too small a factory and marginal costs will be high as the factory is stretched to over produce. Conversely too large a factory results in large fixed costs (eg. air conditioning, or taxes) and low profitability.

Definition 24 Long Run Average Costs: *The minimum cost per unit of producing a given output level when all costs are variable.*

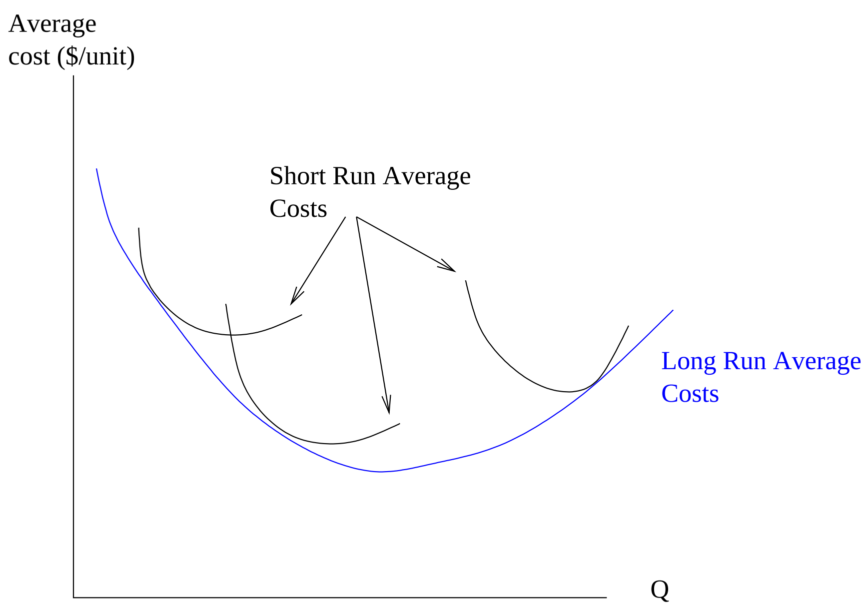

Figure 7: Long run average cost curve.

Long run average costs may be decreasing and then increasing, but also may be strictly decreasing. Here are some LRAC curves for some industries.

1. **Nursing Homes have decreasing LRAC.** Nursing homes have many fixed management costs. Further, larger nursing homes are able to negotiate lower prices for many raw materials.

2. **Cruise Ships have decreasing LRAC.** Huge cruise ships have lower average costs than small cruise ships, economizing on many services provided on the ships.

3. **Hospitals have U-shaped LRAC.** Cowing and Holtmann (1983) in fact found many hospitals in New York should in the long run reduce both capital and physicians to lower average costs.

When the LRAC curve is decreasing, it is often in the interest of the industry to consolidate. A merger with another firm can increase the customer base but reduce the cost per unit, thus increasing profits. Reasons for decreasing long run average costs include specialization of labor and indivisibilities (same as increasing returns to scale).

One reason for increasing costs is that regulation may exempt smaller firms. Another reason for increasing long run costs is coordination and information problems. In a large firm, many individuals do not meaningfully affect profits, and thus have the wrong incentives. Smaller operations may know their customers and production processes better. In this case, spin-offs and divestments may be optimal.

A compromise is franchising. Nationalize just the parts for which increasing returns works.

V Application of long run average costs: Banking mergers

Consider two banks. The first bank services $Q = 15$ customers and the second (smaller) bank services $Q = 5$ customers. The long run average cost function in the industry is:

$$LRAC = 700 - 40Q + Q^2 \tag{80}$$

Revenue is constant at $300 dollars of loan revenue per customer.

Should these two firms merge? The size of the customer base (Q) which minimizes long run average costs is:

$$\min LRAC = 700 - 40Q + Q^2 \tag{81}$$

$$-40 + 2Q = 0 \quad \rightarrow \quad Q = 20 \tag{82}$$

Costs per unit fall until $Q = 20$. Thus these two firms can reduce costs by merging from two firms of size $Q = 15$ and $Q = 5$) into one firm with $Q = 20$.

Profit *per unit* in each case are:

$$\pi = 300 - \left(700 - 40Q + Q^2\right) = -400 + 40Q - Q^2 \tag{83}$$

$$\pi\left(Q = 15\right) = -400 + 40 \cdot 15 - 15^2 = -25 \tag{84}$$

$$\pi\left(Q=5\right)=-400+40\cdot 5+5^{2}=-225 \tag{85}$$

$$\pi\left(Q=20\right)=-400+40\cdot 20-20^{2}=0 \tag{86}$$

Individually, the two banks lose money but together they break even.

VI Measuring Cost Functions

We use the same procedure as with production functions: Obtain data on total costs and quantity produced, and use Excel to fit the data. Both total cost and total quantity produced may appear to be easier to obtain than input data. However, one must remember that costs represent opportunity costs, which are not always straightforward.

Some additional issues:

A Choice of Cost Function

One choice is whether to use a linear, quadratic, or cubic function:

$$TC=a+bQ \tag{87}$$

$$TC=a+bQ+cQ^{2} \tag{88}$$

$$TC=a+bQ+cQ^{2}+dQ^{3} \tag{89}$$

Under most circumstances, the linear cost function does a reasonable job over a narrow range of Q (for example in the short run), but the quadratic and cubic terms must matter theoretically, especially for a wider range of Q. A good strategy might therefore be to estimate the cubic or quadratic.

If the t-stats are low for the quadratic and cubic terms, then predictions are likely to be unreliable for Q that falls outside the data. This indicates using some caution before, for example, committing to large mergers. The following graph illustrates the problem.

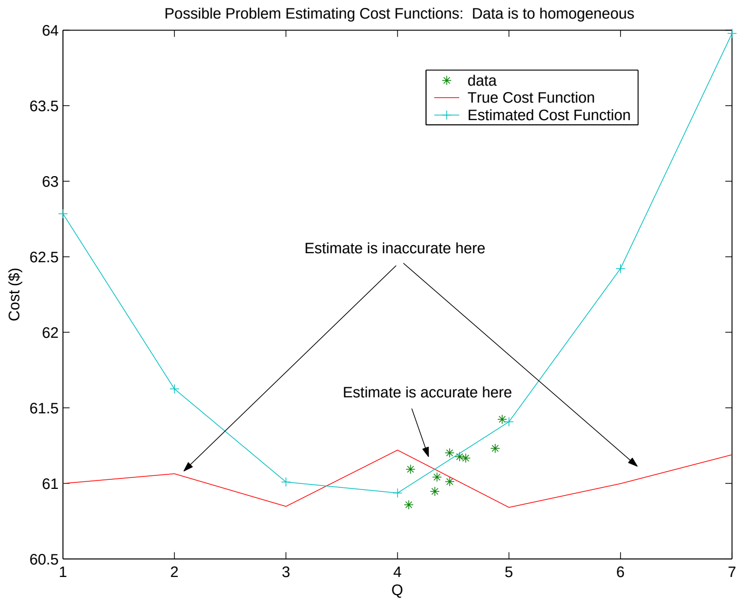

Figure 8: Cost estimation problems.

B Data issues

Some problems with the data that often need correcting:

1. Definition of cost: as mentioned earlier, we use opportunity costs not accounting costs.

2. Price level changes: Historical data is likely to be inaccurate if the price of some inputs or outputs have changed dramatically.

3. What costs vary with output: Some costs have a very limited relationship with output. For example, the number of professionals required may vary in some limited way with output. A firm with $1 million in sales may have two accountants. The firm can obviously increase output to some degree without needing more accountants (so the cost would be fixed). But for larger Q additional accountants are needed (like a variable cost).

4. The cost data needs to match the output data. Often the cost of producing some output may be accounted for in some other period.

5. The firm's technology may change over time.

When estimating long run costs, it is usually preferable to use a cross section of firms across an industry. An individual firm is unlikely to have changed size significantly enough to generate data for a wide range of Q.